Stochastic Model of Vertically Nonhomogeneous Gusts

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Nomenclature

z	= altitude above sea level
z_r	= reference altitude
$u(z),v(a)$	= zonal and meridional gust velocity components
X	= u or v
$L(z)$	= gust length scale
$\sigma(z)$	= gust standard deviation
ξ, ζ	= u/σ_u, v/σ_v
R	= covariance function
t	= $z/L(z)$
τ	= $z_r/L(z_r)-z/L(z)$
$< >$	= ensemble average

Theme

THE small-scale, horizontal gust structure of detailed wind profiles along the vertical, in the first 20 km of the atmosphere, is a vertically nonhomogeneous process. A linear stochastic model is developed for the process based on the process covariance function. This model is formulated using a scaling hypothesis which transforms the nonhomogeneous gust process into a nondimensional gust process which is homogeneous in a nondimensional height coordinate. The velocity scaling parameter for the gust process is the gust standard deviation, and the length scale used to nondimensionalize the altitude is the vertical space lag associated with the first zero of the gust covariance function.

Contents

The most striking characteristic of a vertical profile of the horizontal wind in the first 20 km of the atmosphere is its stochastic spatial variability. All vertical scales of motion, from a few millimeters up to tens of kilometers, are included in these profiles. To design space vehicles with sufficient structural and control system capability to withstand the stochastic "forcing" caused by the detailed structure of these profiles (wind gusts), a model of wind gusts which captures the stochastic character of the wind profile and yet, at the same time, is easily applied is required. The scales of motion associated with vertical wavelength $\lambda < 2000$ *m* are responsible for the high-frequency excitations of the structural and control system modes of response of space vehicles.[1] The gusts associated with these small-scale wind profile features are the subject of this Synoptic.

It is a well-established fact that the wind structure along the vertical in the troposphere and lower stratosphere is not statistically homogeneous. Both experimental[2] and theoretical[3] evidence indicates this nonhomogeneous character. Nonhomogeneity of the wind profile gust statistics, and hence spectra, should be the normal state of affairs in view of the obvious vertical nonhomogeneous structure of atmospheric thermodynamic variables and associated synoptic (weather map) scale wind fields. Accordingly, the goal of this research is to provide a nonhomogeneous stochastic model of the horizontal gusts on the vertical profile of the wind which have wavelengths $\lambda < 2000$ m. The main approach is to model the correlation function. This will necessarily involve a transformation of the vertically nonhomogeneous random zonal and meridional gust processes $u(z)$ and $v(z)$ into the homogeneous (we hope) processes $\xi(t)$ and $\zeta(t)$.

Empirical gust statistics: to develop a stochastic model of horizontal wind gusts for $\lambda < 2000$ m, a sample of detailed wind profile observations measured at 25-m intervals with the FPS-16/Jimsphere wind measurement system[4] was selected for analysis. Each wind profile observation consisted of a zonal and a meridional wind profile in the 1-18 km altitude band. The total sample consisted of approximately 150 wind profile observations for each of the months January, February, and March. The zonal and meridional wind profiles for each observation were digitally filtered with high-pass Martin-Grahm filters, specially developed by DeMandel and Krivo[5] for detailed wind profile analysis, to obtain the zonal and meridional gust profiles, $u(z)$ and $v(z)$, respectively.

The empirical two-point covariance functions

$$R_x(z_r,z)=\left\langle \frac{x(z_r)}{\sigma_x(z_r)}\,\frac{x(z)}{\sigma_x(z)}\right\rangle \tag{1}$$

were determined for z_r incremented at 1000-m intervals and z incremented at 25-m internals relative to altitude z_r for $z<z_r$. Figure 1 contains the experimental estimates of R_u vs nondimensional space lag $(z_r-z)/L(z_r)$, where $l(z_r)$ is the value of z_r-z associated with the first zero of R_u. The experimental estimates of R_v are essentially the same as those of R_u. It was

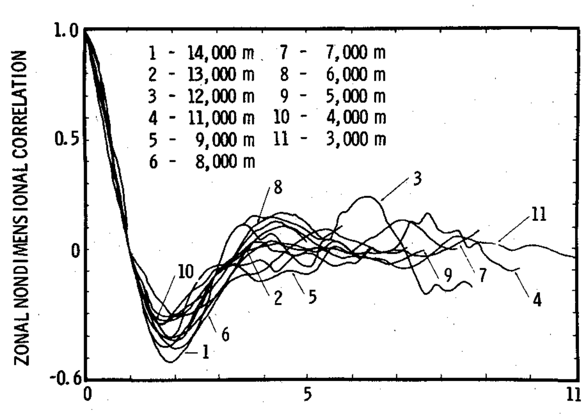

Fig. 1 Covariance function for the zonal component of the gust velocity vector.

Received Oct. 7, 1975; synoptic received March 4, 1976, revision received April 28, 1976. Full paper available from National Technical Information Service, Springfield, Va., 22151 at the standard price (available upon request).

Index categories: LV/M Gust Loading and Wind Shear; LV/M Dynamics and Control; LV/M Structural Design (including Loads

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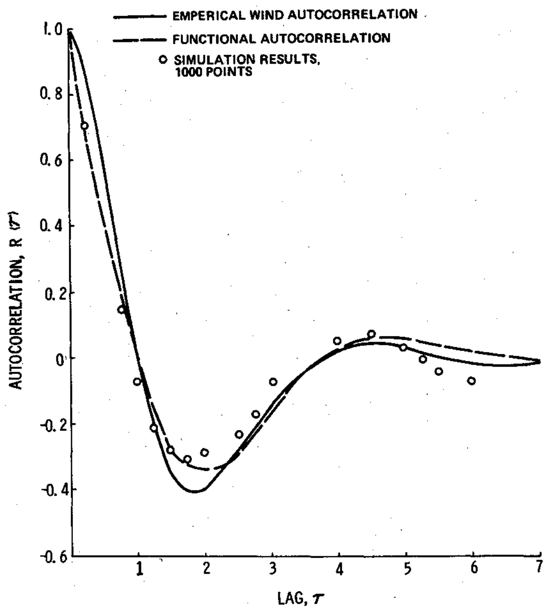

Fig. 2 The autocorrelation function $R[\tau]$.

concluded that the statistics of u and v are equal and therefore they were pooled, to derive the empirical model presented herein. This is physically reasonable. The following formulas summarize the σ and L data

$$\sigma(z)=1.3077\,\mathrm{m\,sec^{-1}},\ z<9160\,\mathrm{m} \tag{2a}$$

$$\sigma(z)=0.346\exp(1.45\times10^{-4}z),\ z\geq9160\,\mathrm{m} \tag{2b}$$

$$L(z)=310+0.0129z,\ z<0160\,\mathrm{m} \tag{3a}$$

$$L(z)=428\,\mathrm{m},\ z\geq9160\,\mathrm{m} \tag{3b}$$

where all units are in the MKS system.

Autocovariance function: it appears that the scaling of $u(z)$ and $v(z)$ indicated in Eq. (1) and the nondimensionalization of (z_r-z) by division with $L(z_r)$ result in a reasonable collapse of the covariance data and, thus, provide a basis for the development of a statistical "law" or model of detailed wind profile gusts. The experimental data indicates that $L(z_r)-L(z)\lesssim0.1L(z)$, so that

$$\frac{z_r}{L(z_r)}-\frac{z}{L(z_r)}\approx\frac{z_r}{L(z_r)}-\frac{z}{L(z)} \tag{4}$$

Thus, the data in Fig. 1 appear to be indicating that a length scale $L_0(z)$ exists such that the random process $\xi(t)$ and $\zeta(t)$ are homogeneous processes relative to the coordinate $t=z/L_0(z)$, where $L(z)$ is an estimate of $L_0(z)$. This conjecture is the basis for the development that follows.

The empirical autocovariances can be approximated in functional form by

$$R(\tau)=\exp(-D|\tau|)\left\{\cos(\mathrm{B}|\tau|)-\frac{\mathrm{D}}{\mathrm{B}}\sin(\mathrm{B}|\tau|)\right\} \tag{5}$$

where B and D are nondimensional constants equal to 1.122 and 0.539, respecitvely. A comparison of the empirical and functional forms of the autocovariance given in Fig. 2 shows good agreement between the two.

Discussion of results: the model presented herein is applicable to a number of important launch vehicle design problems. One major problem is the simulation of gust profiles for launch vehicle flight simulation on digital and analog computers. To perform this simulation, Gaussian white noise is passed through a filter in t-space. The filter can be derived by factoring the spectrum associated with the correlation function $R(\tau)$ [see Eq. (5)]. This procedure yields the homogeneous random process $\xi(t)$ and $\zeta(t)$. Transformation of these processes to the nonhomogeneous processes $u(z)$ and $v(z)$ is obtained by applying Eqs. (2) and (3). However, it should be kept in mind that the wind data used to develop the stochastic model here were digitized at 25-m intervals along the vertical, and that the data processing procedures used to derive the wind profiles from the FPS-16 tracking data, were designed so that nearly all (~90%) of the spectral energy of the gust process existed at wavelength $\lambda=100$ m with zero spectral energy at $\lambda=50$ m.[6] This means the Nyquist wavelength is 50 m, which corresponds to a vertical digitization interval $\Delta z\approx25$ m in z-space. Since the typical value of $L=400$ m, then the digitization interval T of the input noise process in t-space should be no larger than $T\approx\Delta z/L\approx0.06$. Figure 2 gives the results of a 1000-point simulation with $T=0.06$ and is in good agreement with the desired autocorrelation. Usually, measured wind profile samples are used for final design verification. Samples of wind profiles for the first 18 km of the stmosphere with sufficient gust detail for space vehicle design studies are available for Cape Canaveral, Fla.[7] These samples consist of 150 detailed wind profiles for each month measured with the FPS-16 Radar/Jimsphere system.[4] A program is now under way to collect sufficient data to prepare a similar collection for the Vandenberg Air Force Base to be used in design verification studies for the Space Shuttle. This collection will not be available until late 1978, so that a collection must be made available via simulation schemes as previously described. The gust model presented here has been used to develop a wind profile ensemble with available rawinsonde[7] data. To generate this detailed wind profile ensemble, the short wavelength ($\lambda\leq2000$ m) information that is not contained in the rawinsonde data will be simulated with the stochastic gust model described here. Approximately 12 years of rawinsonde wind data are available for the Vandenberg Air Force Base, sufficient to generate the needed detailed wind profile ensemble.

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